# Online Appendix for <br> "Safety, Liquidity, and the Natural Rate of Interest" 

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## A Gibbs Sampler for VARs with Common Trends

Let use the notation $x_{i: j}$ to denote the sequence $\left\{x_{i}, . ., x_{j}\right\}$ for a generic variable $x_{t}$. The Gibbs sampler is structured according to the following blocks:

1. $\bar{y}_{0: T}, \tilde{y}_{-p+1: T}, \lambda \mid \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}$
(a) $\lambda \mid \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}$
(b) $\bar{y}_{0: T}, \tilde{y}_{-p+1: T} \mid \lambda, \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}$
2. $\varphi, \Sigma_{\varepsilon}, \Sigma_{e} \mid \bar{y}_{0: T}, \tilde{y}_{-p+1: T}, \lambda, y_{1: T}$
(a) $\Sigma_{\varepsilon}, \Sigma_{e} \mid \bar{y}_{0: T}, \tilde{y}_{-p+1: T}, \lambda, y_{1: T}$
(b) $\varphi \mid \Sigma_{\varepsilon}, \Sigma_{e}, \bar{y}_{0: T}, \tilde{y}_{-p+1: T}, \lambda, y_{1: T}$

Details of each step follow:

1. $\bar{y}_{0: T}, \tilde{y}_{-p+1: T}, \lambda \mid \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}$

This is given by the product of the marginal posterior distribution of $\lambda$ (conditional on the other parameters) times the distribution of $\bar{y}_{0: T}, \tilde{y}_{-p+1: T}$ conditional on $\lambda$ (and the other parameters).
(a) $\lambda \mid \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}$

The marginal posterior distribution of $\lambda$ (conditional on the other parameters) is given by

$$
p\left(\lambda \mid \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}\right) \propto L\left(y_{1: T} \mid \lambda, \varphi, \Sigma_{\varepsilon}, \Sigma_{e}\right) p(\lambda)
$$

where $L\left(y_{1: T} \mid \lambda, \varphi, \Sigma_{\varepsilon}, \Sigma_{e}\right)$ is the likelihood obtained from the Kalman filter applied to the state space system (2) through (6). $p\left(\lambda \mid \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}\right)$ does not have a known form so we will use a Metropolis Hastings step.
(b) $\bar{y}_{0: T}, \tilde{y}_{-p+1: T} \mid \lambda, \varphi, \Sigma_{\varepsilon}, \Sigma_{e}, y_{1: T}$

Given $\lambda$ and the other parameters of the state space model we can use Durbin and Koopman (2002)'s simulation smoother to obtain draws for the latent states $\bar{y}_{0: T}$ and $\tilde{y}_{-p+1: T}$. Note that in addition to $\bar{y}_{1: T}$ and $\tilde{y}_{1: T}$ we also need to draw the initial conditions $\bar{y}_{0}$ and $\tilde{y}_{-p+1: 0}$ in order to estimate the parameters of (4) and (3) in the next Gibbs sampler step.

Note that missing observations do not present any difficulty in terms of carrying out this step: if the vector $y_{t_{0}}$ has some missing elements, the corresponding rows of the observation equation (2) are simply deleted for $t=t_{0}$.
2. $\varphi, \Sigma_{\varepsilon}, \Sigma_{e} \mid \bar{y}_{0: T}, \tilde{y}_{-p+1: T}, \lambda, y_{1: T}$

This step is straightforward because for given $\bar{y}_{0: T}$ and $\tilde{y}_{-p+1: T}$ equations (3) and (4) are standard VARs where in case of (3) we actually know the autoregressive matrices. The posterior distribution of $\Sigma_{e}$ is given by

$$
p\left(\Sigma_{e} \mid \bar{y}_{0: T}\right)=\mathcal{I} \mathcal{W}\left(\underline{\Sigma}_{e}+\hat{S}_{e}, \kappa_{e}+T\right)
$$

where $\hat{S}_{e}=\sum_{t=1}^{T}\left(\bar{y}_{t}-\bar{y}_{t-1}\right)\left(\bar{y}_{t}-\bar{y}_{t-1}\right)^{\prime}$. The posterior distribution of $\varphi$ and $\Sigma_{\varepsilon}$ is given by

$$
\begin{aligned}
& p\left(\Sigma_{\varepsilon} \mid \tilde{y}_{0: T}\right)=\mathcal{I} \mathcal{W}\left(\underline{\Sigma}_{\varepsilon}+\hat{S}_{\varepsilon}, \kappa_{\varepsilon}+T\right) \\
& p\left(\varphi \mid \Sigma_{\varepsilon}, \tilde{y}_{0: T}\right)=\mathcal{N}\left(\operatorname{vec}(\hat{\Phi}), \Sigma_{\varepsilon} \otimes\left(\sum_{t=1}^{T} \tilde{x}_{t} \tilde{x}_{t}^{\prime}+\underline{\Omega}^{-1}\right)^{-1}\right),
\end{aligned}
$$

where $\tilde{x}_{t}=\left(\tilde{y}_{t-1}^{\prime}, . ., \tilde{y}_{t-p}^{\prime}\right)^{\prime}$ collects the VAR regressors,

$$
\hat{\Phi}=\left(\sum_{t=1}^{T} \tilde{x}_{t} \tilde{x}_{t}^{\prime}+\underline{\Omega}^{-1}\right)^{-1}\left(\sum_{t=1}^{T} \tilde{x}_{t} \tilde{y}_{t}^{\prime}+\underline{\Omega}^{-1} \underline{\Phi}\right), \hat{S}_{\varepsilon}=\sum_{t=1}^{T} \hat{\varepsilon}_{t} \hat{\varepsilon}_{t}^{\prime}+(\hat{\Phi}-\underline{\Phi})^{\prime} \underline{\Omega}^{-1}(\hat{\Phi}-\underline{\Phi})
$$

and $\hat{\varepsilon}_{t}=\tilde{y}_{t}-\hat{\Phi}^{\prime} \tilde{x}_{t}$ are the VAR residuals.

We use 100,000 draws and discard the first 50,000.

## B DSGE Model (Section III)

This section describes the model specification, the data used, how they relate to the model concepts, and the priors distributions assumed for estimation.

The model economy is populated by eight classes of agents: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes. We solve each agent's problem, and derive the resulting equilibrium conditions, which we approximate around the non-stochastic steady state. Since the derivation follows closely the literature (e.g., Christiano et al. (2005)), we describe here the log-linearized conditions.

Growth in the economy is driven by technological progress, $Z_{t}^{*}=e^{\frac{1}{1-\alpha} \tilde{z}_{t}} Z_{t}^{p} e^{\gamma t}$, which is assumed to include a deterministic trend $\left(e^{\gamma t}\right)$, a stochastic trend $\left(Z_{t}^{p}\right)$, and a stationary component $\left(\tilde{z}_{t}\right)$, where $\alpha$ is the income share of capital (after paying mark-ups and fixed costs in production). Trending variables are divided by $Z_{t}^{*}$ to express the model's equilibrium conditions in terms of the stationary variables. In what follows, all variables are expressed in $\log$ deviations from their steady state, and steady-state values are denoted by $*$-subscripts.

The stationary component of productivity $\tilde{z}_{t}$ and the growth rate of the stochastic trend $z_{t}^{p}=\log \left(Z_{t}^{p} / Z_{t-1}^{p}\right)$ are assumed to follow $\mathrm{AR}(1)$ processes:

$$
\begin{gather*}
\tilde{z}_{t}=\rho_{z} \tilde{z}_{t-1}+\sigma_{z} \varepsilon_{z, t}, \varepsilon_{z, t} \sim N(0,1) .  \tag{A-1}\\
z_{t}^{p}=\rho_{z^{p}} z_{t-1}^{p}+\sigma_{z^{p}} \epsilon_{z^{p}, t}, \epsilon_{z^{p}, t} \sim N(0,1) . \tag{A-2}
\end{gather*}
$$

The growth rate of technology evolves thus according to

$$
\begin{equation*}
z_{t} \equiv \log \left(Z_{t}^{*} / Z_{t-1}^{*}\right)-\gamma=\frac{1}{1-\alpha}\left(\tilde{z}_{t}-\tilde{z}_{t-1}\right)+z_{t}^{p} \tag{A-3}
\end{equation*}
$$

where $\gamma$ is the steady-state growth rate of the economy.

The optimal allocation of consumption satisfies the following Euler equation:

$$
\begin{align*}
& c_{t}=-\frac{1-\bar{h}}{\sigma_{c}(1+\bar{h})}\left(R_{t}-\mathbb{E}_{t}\left[\pi_{t+1}\right]+c y_{t}\right)+\frac{\bar{h}}{1+\bar{h}}\left(c_{t-1}-z_{t}\right) \\
&+\frac{1}{1+\bar{h}} \mathbb{E}_{t}\left[c_{t+1}+z_{t+1}\right]+\frac{\left(\sigma_{c}-1\right)}{\sigma_{c}(1+\bar{h})} \frac{w_{*} L_{*}}{c_{*}}\left(L_{t}-\mathbb{E}_{t}\left[L_{t+1}\right]\right) \tag{A-4}
\end{align*}
$$

where $c_{t}$ is consumption, $L_{t}$ denotes hours worked, $R_{t}$ is the nominal interest rate, and $\pi_{t}$ is inflation. The parameter $\sigma_{c}$ captures the degree of relative risk aversion while $\bar{h} \equiv h e^{-\gamma}$ depends on the degree of habit persistence in consumption, $h$, and steady-state growth. This equation includes hours worked because utility is non-separable in consumption and leisure.

The convenience yield $c y_{t}$ contains both a liquidity component $c y_{t}^{l}$ and a safety component $c y_{t}^{s}$

$$
\begin{equation*}
c y_{t}=c y_{t}^{l}+c y_{t}^{s} \tag{A-5}
\end{equation*}
$$

where we let each premium be given by the sum of two $\operatorname{AR}(1)$ processes, one that captures highly persistent movements $\left(c y_{t}^{P, l}\right.$ and $c y_{t}^{P, s}$ ) with autoregressive coefficients fixed at .99, and one that captures transitory fluctuations $\left(\tilde{y}_{t}^{P, l}\right.$ and $\left.\tilde{c y}_{t}^{P, s}\right)$.

The optimal investment decision satisfies the following relationship between the level of investment $i_{t}$, measured in terms of consumption goods, and the value of capital in terms of consumption $q_{t}^{k}$ :

$$
\begin{equation*}
i_{t}=\frac{q_{t}^{k}}{S^{\prime \prime} e^{2 \gamma}(1+\bar{\beta})}+\frac{1}{1+\bar{\beta}}\left(i_{t-1}-z_{t}\right)+\frac{\bar{\beta}}{1+\bar{\beta}} \mathbb{E}_{t}\left[i_{t+1}+z_{t+1}\right]+\mu_{t} . \tag{A-6}
\end{equation*}
$$

This relationship shows that investment is affected by investment adjustment costs ( $S^{\prime \prime}$ is the second derivative of the adjustment cost function) and by an exogenous process $\mu_{t}$, which we call "marginal efficiency of investment", that alters the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The shock $\mu_{t}$ follows an $\operatorname{AR}(1)$ process with parameters $\rho_{\mu}$ and $\sigma_{\mu}$. The parameter $\bar{\beta} \equiv \beta e^{\left(1-\sigma_{c}\right) \gamma}$ depends on the intertemporal discount rate in the household utility function, $\beta$, on the degree of relative risk aversion $\sigma_{c}$, and on the steady-state growth rate $\gamma$.

The capital stock, $\bar{k}_{t}$, which we refer to as "installed capital", evolves as

$$
\begin{equation*}
\bar{k}_{t}=\left(1-\frac{i_{*}}{\bar{k}_{*}}\right)\left(\bar{k}_{t-1}-z_{t}\right)+\frac{i_{*}}{\bar{k}_{*}} i_{t}+\frac{i_{*}}{\bar{k}_{*}} S^{\prime \prime} e^{2 \gamma}(1+\bar{\beta}) \mu_{t}, \tag{A-7}
\end{equation*}
$$

where $i_{*} / \bar{k}_{*}$ is the steady state investment to capital ratio. Capital is subject to variable capacity utilization $u_{t}$; effective capital rented out to firms, $k_{t}$, is related to $\bar{k}_{t}$ by:

$$
\begin{equation*}
k_{t}=u_{t}-z_{t}+\bar{k}_{t-1} \tag{A-8}
\end{equation*}
$$

The optimality condition determining the rate of capital utilization is given by

$$
\begin{equation*}
\frac{1-\psi}{\psi} r_{t}^{k}=u_{t} \tag{A-9}
\end{equation*}
$$

where $r_{t}^{k}$ is the rental rate of capital and $\psi$ captures the utilization costs in terms of foregone consumption.

Real marginal costs for firms are given by

$$
\begin{equation*}
m c_{t}=w_{t}+\alpha L_{t}-\alpha k_{t} \tag{A-10}
\end{equation*}
$$

where $w_{t}$ is the real wage. From the optimality conditions of goods producers it follows that all firms have the same capital-labor ratio:

$$
\begin{equation*}
k_{t}=w_{t}-r_{t}^{k}+L_{t} . \tag{A-11}
\end{equation*}
$$

We include financial frictions in the model, building on the work of Bernanke et al. (1999), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness.

The realized return on capital is given by

$$
\begin{equation*}
\tilde{R}_{t}^{k}-\pi_{t}=\frac{r_{*}^{k}}{r_{*}^{k}+(1-\delta)} r_{t}^{k}+\frac{(1-\delta)}{r_{*}^{k}+(1-\delta)} q_{t}^{k}-q_{t-1}^{k} \tag{A-12}
\end{equation*}
$$

where $\tilde{R}_{t}^{k}$ is the gross nominal return on capital for entrepreneurs, $r_{*}^{k}$ is the steady state value of the rental rate of capital $r_{t}^{k}$, and $\delta$ is the depreciation rate.

The excess return on capital (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the convenience yield $c y_{t}$, the entrepreneurs' leverage (i.e. the ratio of the value of capital to net worth), and "risk shocks" $\tilde{\sigma}_{\omega, t}$ capturing mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)):

$$
\begin{equation*}
E_{t}\left[\tilde{R}_{t+1}^{k}-R_{t}\right]=c y_{t}+\zeta_{s p, b}\left(q_{t}^{k}+\bar{k}_{t}-n_{t}\right)+\tilde{\sigma}_{\omega, t} \tag{A-13}
\end{equation*}
$$

where $n_{t}$ is entrepreneurs' net worth, $\zeta_{s p, b}$ is the elasticity of the credit spread to the entrepreneurs' leverage $\left(q_{t}^{k}+\bar{k}_{t}-n_{t}\right)$. $\tilde{\sigma}_{\omega, t}$ follows an $\operatorname{AR}(1)$ process with parameters $\rho_{\sigma_{\omega}}$ and $\sigma_{\sigma_{\omega}}$. Entrepreneurs' net worth $n_{t}$ evolves in turn according to

$$
\begin{align*}
n_{t}= & \zeta_{n, \tilde{R}^{k}}\left(\tilde{R}_{t}^{k}-\pi_{t}\right)-\zeta_{n, R}\left(R_{t-1}-\pi_{t}+c y_{t-1}\right)+\zeta_{n, q K}\left(q_{t-1}^{k}+\bar{k}_{t-1}\right)+\zeta_{n, n} n_{t-1} \\
& -\gamma_{*} \frac{v_{*}}{n_{*}} z_{t}-\frac{\zeta_{n, \sigma_{\omega}}}{\zeta_{s p, \sigma_{\omega}}} \tilde{\sigma}_{\omega, t-1}, \tag{A-14}
\end{align*}
$$

where the $\zeta$ 's denote elasticities, that depend among others on the entrepreneurs' steadystate default probability $F(\bar{\omega})$, where $\gamma_{*}$ is the fraction of entrepreneurs that survive and continue operating for another period, and where $v_{*}$ is the entrepreneurs' real equity divided by $Z_{t}^{*}$, in steady state.

The production function is

$$
\begin{equation*}
y_{t}=\Phi_{p}\left(\alpha k_{t}+(1-\alpha) L_{t}\right), \tag{A-15}
\end{equation*}
$$

where $\Phi_{p}=1+\Phi / y_{*}$, and $\Phi$ measures the size of fixed costs in production. The resource constraint is:

$$
\begin{equation*}
y_{t}=g_{*} g_{t}+\frac{c_{*}}{y_{*}} c_{t}+\frac{i_{*}}{y_{*}} i_{t}+\frac{r_{*}^{k} k_{*}}{y_{*}} u_{t} . \tag{A-16}
\end{equation*}
$$

where $g_{t}=\log \left(\frac{G_{t}}{Z_{t}^{*} y_{*} g_{*}}\right)$ and $g_{*}=1-\frac{c_{*}+i_{*}}{y_{*}}$. Government spending $g_{t}$ is assumed to follow the exogenous process:

$$
g_{t}=\rho_{g} g_{t-1}+\sigma_{g} \varepsilon_{g, t}+\eta_{g z} \sigma_{z} \varepsilon_{z, t}
$$

Optimal decisions for price and wage setting deliver the price and wage Phillips curves, which are respectively:

$$
\begin{equation*}
\pi_{t}=\kappa m c_{t}+\frac{\iota_{p}}{1+\iota_{p} \bar{\beta}} \pi_{t-1}+\frac{\bar{\beta}}{1+\iota_{p} \bar{\beta}} \mathbb{E}_{t}\left[\pi_{t+1}\right]+\lambda_{f, t} \tag{A-17}
\end{equation*}
$$

and

$$
\begin{align*}
w_{t}=\frac{\left(1-\zeta_{w} \bar{\beta}\right)\left(1-\zeta_{w}\right)}{(1+\bar{\beta}) \zeta_{w}\left(\left(\lambda_{w}-1\right) \epsilon_{w}+1\right)}\left(w_{t}^{h}-w_{t}\right) & -\frac{1+\iota_{w} \bar{\beta}}{1+\bar{\beta}} \pi_{t}+\frac{1}{1+\bar{\beta}}\left(w_{t-1}-z_{t}+\iota_{w} \pi_{t-1}\right) \\
& +\frac{\bar{\beta}}{1+\bar{\beta}} \mathbb{E}_{t}\left[w_{t+1}+z_{t+1}+\pi_{t+1}\right]+\lambda_{w, t}, \tag{A-18}
\end{align*}
$$

where $\kappa=\frac{\left(1-\zeta_{p} \bar{\beta}\right)\left(1-\zeta_{p}\right)}{\left(1+\iota_{p} \bar{\beta}\right) \zeta_{p}\left(\left(\Phi_{p}-1\right) \epsilon_{p}+1\right)}$, the parameters $\zeta_{p}, \iota_{p}$, and $\epsilon_{p}$ are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices,
and $\zeta_{w}, \iota_{w}$, and $\epsilon_{w}$ are the corresponding parameters for wages. $w_{t}^{h}$ measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$
\begin{equation*}
w_{t}^{h}=\frac{1}{1-\bar{h}}\left(c_{t}-\bar{h} c_{t-1}+\bar{h} z_{t}\right)+\nu_{l} L_{t} \tag{A-19}
\end{equation*}
$$

where $\nu_{l}$ characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups $\lambda_{f, t}$ and $\lambda_{w, t}$ follow the exogenous ARMA $(1,1)$ processes:

$$
\lambda_{f, t}=\rho_{\lambda_{f}} \lambda_{f, t-1}+\sigma_{\lambda_{f}} \varepsilon_{\lambda_{f}, t}-\eta_{\lambda_{f}} \sigma_{\lambda_{f}} \varepsilon_{\lambda_{f}, t-1},
$$

and

$$
\lambda_{w, t}=\rho_{\lambda_{w}} \lambda_{w, t-1}+\sigma_{\lambda_{w}} \varepsilon_{\lambda_{w}, t}-\eta_{\lambda_{w}} \sigma_{\lambda_{w}} \varepsilon_{\lambda_{w}, t-1} .
$$

Finally, the monetary authority follows a policy feedback rule:

$$
\begin{align*}
R_{t}= & \rho_{R} R_{t-1}+\left(1-\rho_{R}\right)\left(\psi_{1}\left(\pi_{t}-\pi_{t}^{*}\right)+\psi_{2}\left(y_{t}-y_{t}^{*}\right)\right)  \tag{A-20}\\
& +\psi_{3}\left(\left(y_{t}-y_{t}^{*}\right)-\left(y_{t-1}-y_{t-1}^{*}\right)\right)+r_{t}^{m}
\end{align*}
$$

where $\pi_{t}^{*}$ is a time-varying inflation target, $y_{t}^{*}$ is a measure of the "full-employment level of output," and $r_{t}^{m}$ captures exogenous departures from the policy rule.

The time-varying inflation target $\pi_{t}^{*}$ is meant to capture the rise and fall of inflation and interest rates in the estimation sample. ${ }^{1}$ As in Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011), we use data on long-run inflation expectations in the estimation of the model. This allows us to pin down the target inflation rate to the extent that long-run inflation expectations contain information about the central bank's objective. The timevarying inflation target evolves according to

$$
\begin{equation*}
\pi_{t}^{*}=\rho_{\pi^{*}} \pi_{t-1}^{*}+\sigma_{\pi^{*}} \epsilon_{\pi^{*}, t}, \tag{A-21}
\end{equation*}
$$

where $0<\rho_{\pi^{*}}<1$ and $\epsilon_{\pi^{*}, t}$ is an iid shock. We model $\pi_{t}^{*}$ as a stationary process, although our prior for $\rho_{\pi^{*}}$ will force this process to be highly persistent.

The "full-employment level of output" $y_{t}^{*}$ represents the level of output that would obtain if prices and wages were fully flexible and if there were no markup shocks. This variable along with the natural rate of interest $r_{t}^{*}$ are obtained by solving the model without nominal

[^0]rigidities and markup shocks (that is, equations (A-4) through (A-19) with $\zeta_{p}=\zeta_{w}=0$, and $\lambda_{f, t}=\lambda_{w, t}=0$ ).

The exogenous component of the policy rule $r_{t}^{m}$ evolves according to the following process

$$
\begin{equation*}
r_{t}^{m}=\rho_{r^{m}} r_{t-1}^{m}+\epsilon_{t}^{R}+\sum_{k=1}^{K} \epsilon_{k, t-k}^{R} \tag{A-22}
\end{equation*}
$$

where $\epsilon_{t}^{R}$ is the usual contemporaneous policy shock, and $\epsilon_{k, t-k}^{R}$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule $k$ periods later, that is, at time $t$. We assume that $\epsilon_{k, t-k}^{R} \sim N\left(0, \sigma_{k, r}^{2}\right)$, i.i.d. As argued in Laseen and Svensson (2011), such anticipated policy shocks allow us to capture the effects of the zero lower bound on nominal interest rates, as well as the effects of forward guidance in monetary policy.

## B. 1 State Space Representation and Data

We use the method in Sims (2002) to solve the system of log-linear approximate equilibrium conditions and obtain the transition equation, which summarizes the evolution of the vector of state variables $s_{t}$ :

$$
\begin{equation*}
s_{t}=\mathcal{T}(\theta) s_{t-1}+\mathcal{R}(\theta) \epsilon_{t} \tag{A-23}
\end{equation*}
$$

where $\theta$ is a vector collecting all the DSGE model parameters and $\epsilon_{t}$ is a vector of all structural shocks. The state-space representation of our model is composed of the transition equation (A-23), and a system of measurement equations:

$$
\begin{equation*}
Y_{t}=\mathcal{D}(\theta)+\mathcal{Z}(\theta) s_{t} \tag{A-24}
\end{equation*}
$$

mapping the states into the observable variables $Y_{t}$, which we describe in detail next.
The estimation of the model is based on data on real output growth (including both GDP and GDI measures), consumption growth, investment growth, real wage growth, hours worked, inflation (measured by core PCE and GDP deflators), short- and long- term interest rates, 10-year inflation expectations, market expectations for the federal funds rate up to 6 quarters ahead, Aaa and Baa credit spreads, and total factor productivity growth unadjusted for variable utilization. Measurement equations (A-24) relate these observables to the model
variables as follows:
GDP growth
Consumption growth

$$
=100 \gamma+\left(y_{t}-y_{t-1}+z_{t}\right)+e_{t}^{g d p}-e_{t-1}^{g d p}
$$

Investment growth
$=100 \gamma+\left(c_{t}-c_{t-1}+z_{t}\right)$
Real Wage growth
$=100 \gamma+\left(i_{t}-i_{t-1}+z_{t}\right)$
Hours
$=100 \gamma+\left(w_{t}-w_{t-1}+z_{t}\right)$
Core PCE Inflation
$+L_{t}$
GDP Deflator Inflation
$=\pi_{*}+\pi_{t}+e_{t}^{p c e}$
FFR
$=\pi_{*}+\delta_{\text {gdpdef }}+\gamma_{\text {gdpdef }} * \pi_{t}+e_{t}^{\text {gdpdef }}$
$\mathrm{FFR}_{t, t+j}^{e}$
10y Nominal Bond Yield
$=R_{*}+R_{t}$
$=R_{*}+\mathbb{E}_{t}\left[R_{t+j}\right], j=1, \ldots, 6$
$=R_{*}+\mathbb{E}_{t}\left[\frac{1}{40} \sum_{j=0}^{39} R_{t+j}\right]+e_{t}^{10 y}$
10y Infl. Expectations
$=\pi_{*}+\mathbb{E}_{t}\left[\frac{1}{40} \sum_{j=0}^{39} \pi_{t+j}\right]$
Aaa-20-year Treasury Spread $=c y_{*}^{l}+\mathbb{E}_{t}\left[\frac{1}{80} \sum_{j=0}^{79} c y_{t+j}^{l}\right]+e_{t}^{A a a}$
Baa-20-year Treasury Spread $=c y_{*}^{l}+c y_{*}^{s}+S P_{*}+\mathbb{E}_{t} \frac{1}{80} \sum_{j=0}^{79}\left[\tilde{R}_{t+j+1}^{k}-R_{t+j}\right]+e_{t}^{\text {Baa }}$
TFP growth, demeaned $\quad=z_{t}+\frac{\alpha}{1-\alpha}\left(u_{t}-u_{t-1}\right)+e_{t}^{t f p}$.

All variables are measured in percent. The terms $\pi_{*}$ and $R_{*}$ measure respectively the net steady-state inflation rate and short-term nominal interest rate, expressed in percentage terms, and $\bar{L}$ captures the mean of hours (this variable is measured as an index). We assume that some of the variables are measured with "error," that is, the observed value equals the model implied value plus an $\mathrm{AR}(1)$ exogenous process $e_{t}^{*}$ that can be thought of either measurement errors or some other unmodeled source of discrepancy between the model and the data, as in Boivin and Giannoni (2006). For instance, the terms $e_{t}^{g d p}$ and $e_{t}^{g d i}$ capture measurement error of total output. ${ }^{2}$ Alternatively, for the long-term nominal interest rate,

[^1]the term $e_{t}^{10 y}$ captures fluctuations in term premia not captured by the model.

## B. 2 Inference, Prior and Posterior Parameter Estimates

We estimate the model using Bayesian techniques. This requires the specification of a prior distribution for the model parameters. For most parameters common with Smets and Wouters (2007), we use the same marginal prior distributions. As an exception, we favor a looser prior than Smets and Wouters (2007) for the quarterly steady state inflation rate $\pi_{*}$; it is centered at $0.75 \%$ and has a standard deviation of $0.4 \%$. Regarding the financial frictions, we specify priors for the parameters $S P_{*}, \zeta_{s p, b}, \rho_{\sigma_{\omega}}$, and $\sigma_{\sigma_{\omega}}$, while we fix the parameters corresponding to the steady state default probability and the survival rate of entrepreneurs, respectively. In turn, these parameters imply values for the parameters of (A-14). Information on the priors and posterior mean is provided in Table A1.

## B. 3 Data Construction

Data on real GDP (GDPC), the GDP deflator (GDPDEF), core PCE inflation (PCEPILFE), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) are produced at a quarterly frequency by the Bureau of Economic Analysis, and are included in the National Income and Product Accounts (NIPA). Average weekly hours of production and nonsupervisory employees for total private industries (AWHNONAG), civilian employment (CE16OV), and the civilian non-institutional population (CNP16OV) are produced by the Bureau of Labor Statistics (BLS) at a monthly frequency. The first of these series is obtained from the Establishment Survey, and the remaining from the Household Survey. Both surveys are released in the BLS Employment Situation Summary. Since our models are estimated on quarterly data, we take averages of the monthly data. Compensation per hour for the non-farm business sector (COMPNFB) is obtained from the Labor Productvity and Costs release, and produced by the BLS at a quarterly frequency. The data are transformed following Smets and Wouters (2007), with the exception of the civilian population data, which are filtered using the Hodrick-Prescott filter to remove jumps around census dates. The federal funds rate is obtained from the Federal Reserve Board's H. 15 release at a business day frequency. We take quarterly averages of the annualized daily data by a comment stochastic trend.
and divide by four. Let $\Delta$ denote the temporal difference operator. Then:

$$
\begin{array}{ll}
\text { Output growth } & =100 * \Delta L N((G D P C) / C N P 16 O V) \\
\text { Consumption growth } & =100 * \Delta L N((P C E C / G D P D E F) / C N P 16 O V) \\
\text { Investment growth } & =100 * \Delta L N((F P I / G D P D E F) / C N P 16 O V) \\
\text { Real wage growth } & =100 * \Delta L N(C O M P N F B / G D P D E F) \\
\text { Hours worked } & =100 * L N((A W H N O N A G * C E 16 O V / 100) / C N P 16 O V) \\
\text { GDP Deflator Inflation } & =100 * \Delta L N(G D P D E F) \\
\text { Core PCE Inflation } & =100 * \Delta L N(P C E P I L F E) \\
\text { FFR } & =(1 / 4) * F E D E R A L F U N D S \text { RATE }
\end{array}
$$

Long-run inflation expectations are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters available from the FRB Philadelphia's Real-Time Data Research Center. Long-run inflation expectations (average CPI inflation over the next 10 years) are available from 1991Q4 onward. Prior to 1991Q4, we use the 10-year expectations data from the Blue Chip survey to construct a long time series that begins in 1979Q4. Since the Blue Chip survey reports long-run inflation expectations only twice a year, we treat these expectations in the remaining quarters as missing observations and adjust the measurement equation of the Kalman filter accordingly. Long-run inflation expectations $\pi_{t}^{O, 40}$ are therefore measured as

$$
10 y \text { Infl } \operatorname{Exp}=(10 \text {-year average CPI inflation forecast }-0.50) / 4
$$

where 0.50 is the average difference between CPI and GDP annualized inflation from the beginning of the sample to 1992 . We divide by 4 to express the data in quarterly terms.

We measure Spread as the annualized Moody's Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity. Both series are available from the Federal Reserve Board's H. 15 release. Like the federal funds rate, the spread data are also averaged over each quarter and measured at a quarterly frequency. This leads to:

$$
\text { Spread }=(1 / 4) *(\text { Baa Corporate }-10 \text { year Treasury }) .
$$

Similarly,

$$
10 \mathrm{y} \text { Bond yield }=(1 / 4) *(10 \text { year Treasury }) .
$$

Lastly, TFP growth is measured using John Fernald's TFP growth series, unadjusted for changes in utilization. That series is demeaned, divided by 4 to express it in quarterly
growth rates, and divided by Fernald's estimate of $(1-\alpha)$ to convert it in labor augmenting terms:

TFP growth, demeaned $=(1 / 4) *($ Fernald's TFP growth, unadjusted, demeaned $) /(1-\alpha)$.

## B. 4 DSGE Model Estimates

Table A1: Parameter Estimates

| Parameter | Prior |  |  | Posterior |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Type | Mean | SD | Mean | 90.0\% Lower Band | 90.0\% Upper Band |
| Steady State |  |  |  |  |  |  |
| $100 \gamma$ | N | 0.400 | 0.100 | 0.418 | 0.332 | 0.503 |
| $\alpha$ | N | 0.300 | 0.050 | 0.180 | 0.156 | 0.205 |
| $100\left(\beta^{-1}-1\right)$ | G | 0.250 | 0.100 | 0.268 | 0.147 | 0.387 |
| $\sigma_{c}$ | N | 1.500 | 0.370 | 0.943 | 0.801 | 1.080 |
| $h$ | B | 0.700 | 0.100 | 0.572 | 0.515 | 0.630 |
| $\nu_{l}$ | N | 2.000 | 0.750 | 2.561 | 1.787 | 3.326 |
| $\delta$ | - | 0.025 | 0.000 | 0.025 | 0.025 | 0.025 |
| $\Phi_{p}$ | - | 1.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| $S^{\prime \prime}$ | N | 4.000 | 1.500 | 1.552 | 0.964 | 2.108 |
| $\psi$ | B | 0.500 | 0.150 | 0.639 | 0.506 | 0.767 |
| $\bar{L}$ | N | -45.000 | 5.000 | -47.836 | -50.168 | -45.587 |
| $\lambda_{w}$ | - | 1.500 | 0.000 | 1.500 | 1.500 | 1.500 |
| $\pi_{*}$ | - | 0.500 | 0.000 | 0.500 | 0.500 | 0.500 |
| $g_{*}$ | - | 0.180 | 0.000 | 0.180 | 0.180 | 0.180 |
| Nominal Rigidities |  |  |  |  |  |  |
| $\zeta_{p}$ | B | 0.500 | 0.100 | 0.954 | 0.944 | 0.964 |
| $\zeta_{w}$ | B | 0.500 | 0.100 | 0.965 | 0.958 | 0.973 |
| $\iota_{p}$ | B | 0.500 | 0.150 | 0.220 | 0.096 | 0.348 |
| $\iota_{w}$ | B | 0.500 | 0.150 | 0.794 | 0.682 | 0.902 |
| $\epsilon_{p}$ | - | 10.000 | 0.000 | 10.000 | 10.000 | 10.000 |
| $\epsilon_{w}$ | - | 10.000 | 0.000 | 10.000 | 10.000 | 10.000 |
| Policy |  |  |  |  |  |  |
| $\psi_{1}$ | N | 1.500 | 0.250 | 1.881 | 1.556 | 2.198 |
| $\psi_{2}$ | N | 0.120 | 0.050 | 0.249 | 0.196 | 0.301 |
| $\psi_{3}$ | N | 0.120 | 0.050 | 0.320 | 0.268 | 0.372 |
| $\rho_{R}$ | B | 0.750 | 0.100 | 0.856 | 0.814 | 0.897 |
| $\rho_{r^{m}}$ | B | 0.500 | 0.200 | 0.227 | 0.134 | 0.323 |
| Financial Frictions |  |  |  |  |  |  |
| $F(\bar{\omega})$ | - | 0.030 | 0.000 | 0.030 | 0.030 | 0.030 |
| $S P_{*}$ | G | 1.000 | 0.100 | 1.044 | 0.881 | 1.211 |
| $\zeta_{s p, b}$ | B | 0.050 | 0.005 | 0.048 | 0.039 | 0.055 |
| $\gamma_{*}$ | - | 0.990 | 0.000 | 0.990 | 0.990 | 0.990 |
| $c y_{*}^{s}$ | - | 0.065 | 0.000 | 0.065 | 0.065 | 0.065 |
| $c y_{*}^{l}$ | - | 0.117 | 0.000 | 0.117 | 0.117 | 0.117 |
| Exogenous Processes |  |  |  |  |  |  |
| $\rho_{g}$ | B | 0.500 | 0.200 | 0.986 | 0.974 | 0.998 |
| $\rho_{\mu}$ | B | 0.500 | 0.200 | 0.971 | 0.949 | 0.994 |

Table A1: Parameter Estimates

| Parameter | Prior |  |  | Posterior |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Type | Mean | SD | Mean | 90.0\% Lower Band | 90.0\% Upper Band |
| $\rho_{z^{p}}$ | - | 0.990 | 0.000 | 0.990 | 0.990 | 0.990 |
| $\rho_{z}$ | B | 0.500 | 0.200 | 0.937 | 0.903 | 0.972 |
| $\rho_{c y^{p}, l}$ | - | 0.990 | 0.000 | 0.990 | 0.990 | 0.990 |
| $\rho_{\text {ç }}^{\text {c }, l}$ | B | 0.500 | 0.200 | 0.515 | 0.229 | 0.778 |
| $\rho_{c y^{p}, s}$ | - | 0.990 | 0.000 | 0.990 | 0.990 | 0.990 |
| $\rho_{\text {çj,s }}$ | B | 0.500 | 0.200 | 0.665 | 0.517 | 0.822 |
| $\rho_{\sigma_{\omega}}$ | B | 0.750 | 0.150 | 0.979 | 0.952 | 1.000 |
| $\rho_{\pi^{*}}$ | - | 0.990 | 0.000 | 0.990 | 0.990 | 0.990 |
| $\rho_{\lambda_{f}}$ | B | 0.500 | 0.200 | 0.787 | 0.678 | 0.901 |
| $\rho_{\lambda_{w}}$ | B | 0.500 | 0.200 | 0.331 | 0.086 | 0.562 |
| $\eta_{\lambda_{f}}$ | B | 0.500 | 0.200 | 0.633 | 0.435 | 0.831 |
| $\eta_{\lambda_{w}}$ | B | 0.500 | 0.200 | 0.428 | 0.232 | 0.611 |
| $\eta_{g z}$ | B | 0.500 | 0.200 | 0.429 | 0.125 | 0.708 |
| $\sigma_{g}$ | IG | 0.100 | 2.000 | 2.241 | 2.044 | 2.430 |
| $\sigma_{\mu}$ | IG | 0.100 | 2.000 | 0.529 | 0.320 | 0.756 |
| $\sigma_{z^{p}}$ | IG | 0.100 | 2.000 | 0.062 | 0.048 | 0.075 |
| $\sigma_{z}$ | IG | 0.100 | 2.000 | 0.533 | 0.484 | 0.583 |
| $\sigma_{c y^{p}, l}$ | IG | 0.013 | 100.000 | 0.013 | 0.012 | 0.015 |
| $\sigma_{\tilde{c y}, l}$ | IG | 0.100 | 2.000 | 0.092 | 0.048 | 0.134 |
| $\sigma_{c y^{p}, s}$ | IG | 0.013 | 100.000 | 0.011 | 0.010 | 0.013 |
| $\sigma_{\tilde{c} \tilde{y}, s}$ | IG | 0.100 | 2.000 | 0.133 | 0.090 | 0.173 |
| $\sigma_{\sigma_{\omega}}$ | IG | 0.050 | 4.000 | 0.096 | 0.056 | 0.134 |
| $\sigma_{\pi_{*}}$ | IG | 0.030 | 6.000 | 0.061 | 0.044 | 0.078 |
| $\sigma_{\lambda_{f}}$ | IG | 0.100 | 2.000 | 0.078 | 0.061 | 0.095 |
| $\sigma_{\lambda_{w}}$ | IG | 0.100 | 2.000 | 0.418 | 0.372 | 0.463 |
| $\sigma_{r^{m}}$ | IG | 0.100 | 2.000 | 0.229 | 0.203 | 0.252 |
| $\sigma_{1, r}$ | IG | 0.200 | 4.000 | 0.094 | 0.074 | 0.114 |
| $\sigma_{2, r}$ | IG | 0.200 | 4.000 | 0.089 | 0.069 | 0.108 |
| $\sigma_{3, r}$ | IG | 0.200 | 4.000 | 0.089 | 0.069 | 0.108 |
| $\sigma_{4, r}$ | IG | 0.200 | 4.000 | 0.085 | 0.066 | 0.104 |
| $\sigma_{5, r}$ | IG | 0.200 | 4.000 | 0.087 | 0.067 | 0.106 |
| $\sigma_{6, r}$ | IG | 0.200 | 4.000 | 0.090 | 0.069 | 0.111 |
| Measurement |  |  |  |  |  |  |
| $\delta_{\text {gdpdef }}$ | N | 0.000 | 2.000 | 0.000 | -0.043 | 0.047 |
| $\gamma_{\text {gdpdef }}$ | N | 1.000 | 2.000 | 1.039 | 0.966 | 1.118 |
| $\rho_{\text {gdp }}$ | N | 0.000 | 0.200 | 0.067 | -0.138 | 0.278 |
| $\rho_{g d i}$ | N | 0.000 | 0.200 | 0.945 | 0.907 | 0.986 |
| $\varrho_{g d p}$ | N | 0.000 | 0.400 | -0.133 | -0.759 | 0.461 |
| $\rho_{\text {gdpdef }}$ | B | 0.500 | 0.200 | 0.509 | 0.381 | 0.645 |
| $\rho_{p c e}$ | B | 0.500 | 0.200 | 0.248 | 0.047 | 0.441 |
| $\rho_{\text {Aaa }}$ | B | 0.500 | 0.100 | 0.610 | 0.471 | 0.753 |
| $\rho_{\text {Baa }}$ | B | 0.500 | 0.100 | 0.787 | 0.674 | 0.896 |
| $\rho_{10 y}$ | B | 0.500 | 0.200 | 0.960 | 0.935 | 0.987 |
| $\rho_{t f p}$ | B | 0.500 | 0.200 | 0.178 | 0.070 | 0.279 |

Table A1: Parameter Estimates

| Parameter | Prior |  |  | Posterior |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Type | Mean | SD | Mean | 90.0\% Lower Band | 90.0\% Upper Band |
| $\sigma_{\text {gdp }}$ | IG | 0.100 | 2.000 | 0.251 | 0.210 | 0.293 |
| $\sigma_{g d i}$ | IG | 0.100 | 2.000 | 0.308 | 0.269 | 0.343 |
| $\sigma_{\text {gdpdef }}$ | IG | 0.100 | 2.000 | 0.164 | 0.147 | 0.182 |
| $\sigma_{p c e}$ | IG | 0.100 | 2.000 | 0.099 | 0.081 | 0.118 |
| $\sigma_{\text {Aaa }}$ | IG | 0.100 | 2.000 | 0.024 | 0.020 | 0.027 |
| $\sigma_{B a a}$ | IG | 0.100 | 2.000 | 0.047 | 0.039 | 0.056 |
| $\sigma_{10 y}$ | IG | 0.750 | 2.000 | 0.121 | 0.110 | 0.132 |
| $\sigma_{t f p}$ | IG | 0.100 | 2.000 | 0.744 | 0.676 | 0.811 |

Note: T N, B and G stand, respectively, for Normal, Beta and Gamma distributions. For Inverse Gamma (IG) distributions, we report the coefficients $\tau$ and $\nu$ instead of the prior mean and SD.

## C Additional Tables and Figures - VARs (Section II)

Figure A1: Other Trends and Observables, Baseline Model


Note: The figure shows $R_{80, t}-R_{1, t}$ (dotted blue line) together with the trend $\overline{t p}_{t}$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A2: $y_{t}, \Lambda \bar{y}_{t}$, and $\tilde{y}_{t}$; Baseline Model


Note: For each variable the top panel shows the data $y_{t}$ and the trend component $\Lambda \bar{y}_{t}$, and the bottom panel shows the stationary component $\tilde{y}_{t}$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A3: Other Trends and Observables, Convenience Yield Model



Note: The left panel shows $\pi_{t}$ (dotted blue line), and $\pi_{t}^{e}$ (solid blue line), together with the trend $\bar{\pi}_{t}$. The right panel shows $R_{80, t}-R_{1, t}$ (dotted blue line) together with the trend $\overline{t p}_{t}$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A4: $y_{t}, \Lambda \bar{y}_{t}$, and $\tilde{y}_{t}$; Convenience Yield Model


Note: For each variable the top panel shows the data $y_{t}$ and the trend component $\Lambda \bar{y}_{t}$, and the bottom panel shows the stationary component $\tilde{y}_{t}$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A5: Other Trends and Observables, Safety and Liquidity Model


Note: The top left panel shows $\pi_{t}$ (dotted blue line), and $\pi_{t}^{e}$ (solid blue line), together with the trend $\bar{\pi}_{t}$. The top right panel shows $R_{1, t}-\pi_{t}^{e}$ (dotted blue line), and $R_{1, t}^{e}-\pi_{t}^{e}$ (blue dots), together with the trend $\bar{r}_{t}$. The bottom left panel shows $R_{1, t}-\pi_{t}^{e}-\left(R_{t}^{B a a}-R_{80, t}\right)$ (dotted blue line), together with the trend $\bar{m}_{t}$. The bottom right panel shows $R_{80, t}-R_{1, t}$ (dotted blue line) together with the trend $\overline{t p}_{t}$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A6: Prior and Posterior Distributions of the Standard Deviations of the Shocks to the Trend Components


Note: The panels show the prior (solid red line) and posterior (histogram) distributions of the standard deviations of the shocks to the trend components - the diagonal elements of the matrix $\Sigma_{e}$. The units are expressed in terms of multiples of $1 \%$ per century, that is, $\sqrt{1 / 400}$.

Figure A7: $y_{t}, \Lambda \bar{y}_{t}$, and $\tilde{y}_{t}$; Safety and Liquidity Model




Note: For each variable the top panel shows the data $y_{t}$ and the trend component $\Lambda \bar{y}_{t}$, and the bottom panel shows the stationary component $\tilde{y}_{t}$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A8: Other Trends and Observables, Consumption Growth Model


Note: The top left panel shows $\pi_{t}$ (dotted blue line), and $\pi_{t}^{e}$ (solid blue line), together with the trend $\bar{\pi}_{t}$. The top right panel shows $R_{1, t}-\pi_{t}^{e}$ (dotted blue line), and $R_{1, t}^{e}-\pi_{t}^{e}$ (blue dots), together with the trend $\bar{r}_{t}$. The middle left panel shows $R_{1, t}-\pi_{t}^{e}-\left(R_{t}^{B a a}-R_{80, t}\right)$ (dotted blue line), together with the trend $\bar{m}_{t}$. The middle right panel shows $R_{80, t}-R_{1, t}$ (dotted blue line) together with the trend $\overline{t p}_{t}$. The bottom left panel shows the Baa/Aaa spread $R_{t}^{B a a}-R_{t}^{A a a}$ (dotted blue line), together with the trend $\overline{c y}_{t}^{s}$. The bottom right panel shows the Aaa/Treasury spread $R_{t}^{A a a}-R_{80, t}$ (dotted blue line), together with the trend $\overline{c y}_{t}^{l}$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A9: $y_{t}, \Lambda \bar{y}_{t}$, and $\tilde{y}_{t}$; Consumption Growth Model

















Note: For each variable the top panel shows the data $y_{t}$ and the trend component $\Lambda \bar{y}_{t}$, and the bottom panel shows the stationary component $\tilde{y}_{t}$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

## D Robustness - VAR (Section II)

Table A2: Change in Trends, 1998Q1-2016Q4 - Robustness


Note: The table shows the change in the trends for the different specifications described in section II.D, the model with default (column (1)), loose prior (column (2)), inflation trends in the term spread (column (3)), and labor productivity (column (4)). For each trend, the table shows the posterior median, the 68 (square bracket) and 95 (round bracket) percent posterior coverage intervals. The $* *$ symbol indicates that the decline is significant, in that the 95 percent coverage intervals do not include zero.

Figure A10: Posterior Distribution of $\gamma^{t p}$ - Model with Inflation Affecting the Nominal Term Premium


Note: The figure shows the posterior distribution of $\gamma^{t p}$. The prior is an exponential with mean . 10 .

Figure A11: Trends and Observables, Inflation Affecting the Nominal Term Premium


Note: The top left panel shows $\pi_{t}$ (dotted blue line), and $\pi_{t}^{e}$ (solid blue line), together with the trend $\bar{\pi}_{t}$. The top right panel shows $R_{1, t}-\pi_{t}^{e}$ (dotted blue line), and $R_{1, t}^{e}-\pi_{t}^{e}$ (blue dots), together with the trend $\bar{r}_{t}$. The middle left panel shows $R_{1, t}-\pi_{t}^{e}-\left(R_{t}^{B a a}-R_{80, t}\right)$ (dotted blue line), together with the trend $\bar{m}_{t}$. The middle right panel shows $R_{80, t}-R_{1, t}$ (dotted blue line) together with the trend $\overline{t p}_{t}$. The bottom left panel shows the Baa/Aaa spread $R_{t}^{B a a}-R_{t}^{A a a}$ (dotted blue line), together with the trend $\overline{c y}_{t}^{s}$. The bottom right panel shows the Aaa/Treasury spread $R_{t}^{A a a}-R_{80, t}$ (dotted blue line), together with the trend $\overline{c y}_{t}^{l}$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

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[^0]:    ${ }^{1}$ The assumption that the inflation target moves exogenously is of course a simplification. A more realistic model would for instance relate movements in trend inflation to the evolution of the policy makers' understanding of the output-inflation trade-of, as in Sargent (1999) or Primiceri (2006).

[^1]:    ${ }^{2}$ We introduce correlation in the measurement errors for GDP and GDI, which evolve as follows:

    $$
    \begin{aligned}
    e_{t}^{g d p} & =\rho_{g d p} \cdot e_{t-1}^{g d p}+\sigma_{g d p} \epsilon_{t}^{g d p}, \epsilon_{t}^{g d p} \sim i . i . d . N(0,1) \\
    e_{t}^{g d i} & =\rho_{g d i} \cdot e_{t-1}^{g d i}+\varrho_{g d p} \cdot \sigma_{g d p} \epsilon_{t}^{g d p}+\sigma_{g d i} \epsilon_{t}^{g d i}, \epsilon_{t}^{g d i} \sim i . i . d . N(0,1) .
    \end{aligned}
    $$

    The measurement errors for GDP and GDI are thus stationary in levels, and enter the observation equation in first differences (e.g. $e_{t}^{g d p}-e_{t-1}^{g d p}$ and $e_{t}^{g d p}-e_{t-1}^{g d p}$ ). GDP and GDI are also cointegrated as they are driven

